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Functional Relationship Between Primary and Secondary Delays on Railway Lines

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Functional Relationship Between Primary and Secondary Delays on Railway Lines

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Polynomial Aggregate Delay Function

- Management question
- Problem definition
- Formulation
- Some fun graphical results
- Conclusions recommend general "rule of thumb" timetable guidelines

Strategic Design of Timetable

- Goals
  - Provide service, capacity
  - Minimize travel time
  - Promise punctuality
- Controls
  - Frequency of service
  - Timetable ”slack”
    - Extra train scheduled time
    - Extra separation between adjacent trains

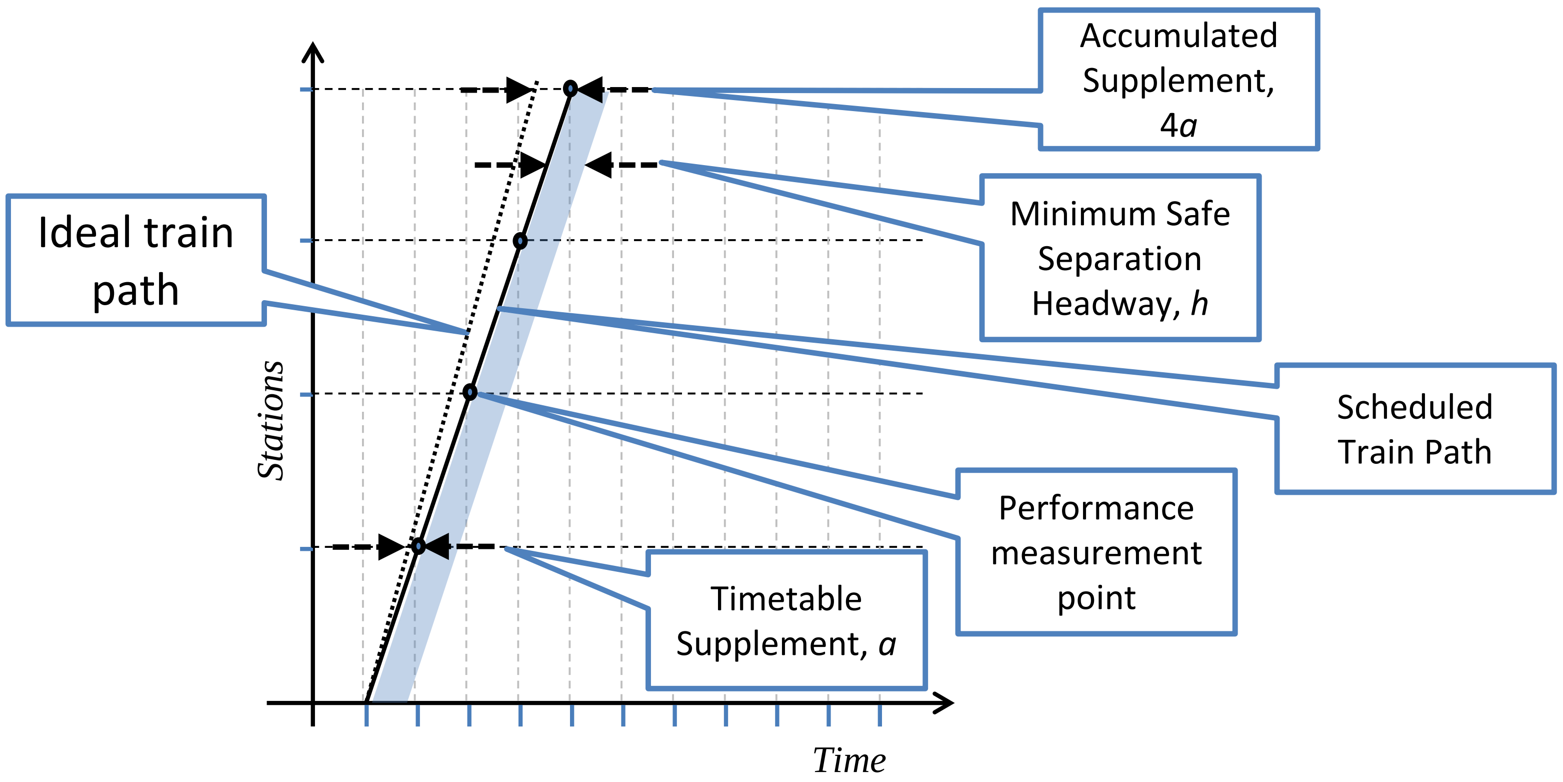
40 Helsingør - København H - Malmö C
Gyldig 8. august 2016 - 10. december 2016

Køredage		①-⑤	①-⑤		
Helsingør	1 7.45	7.52	8.02		1 8.05
Snekkersten	7.49	7.57	8.06		8.09
Espergærde	7.52	8.00	8.09		8.12
Humlebæk	1 7.56	8.04	8.12		1 8.16
Nivå	8.00		8.16		8.20
Kokkedal	8.03	8.11	8.20		8.23
Rungsted Kyst	8.07	8.15			8.27
Vedbæk	8.11				8.31
Skodsborg	8.14				8.34
Klampenborg	8.19				8.39
Hellerup	8.24		8.35		8.44
Østerport	8.30	8.33	8.41		8.50
Nørreport	8.33	8.36	8.44		8.53
København H ○	8.37	8.40	8.48		8.57
København H	8.40				9.00
Ørestad	8.46				9.06
Tårnby	8.49				9.09
CPH Lufthavn ✈ ◆ ○	8.57				9.17
CPH Lufthavn ✈				9.06	
Hyllie ○				9.18	
Hyllie				9.28	
Triangeln				9.31	
Malmö C ○				9.35	
Tognummer	1826	4428	2028	29728	1828

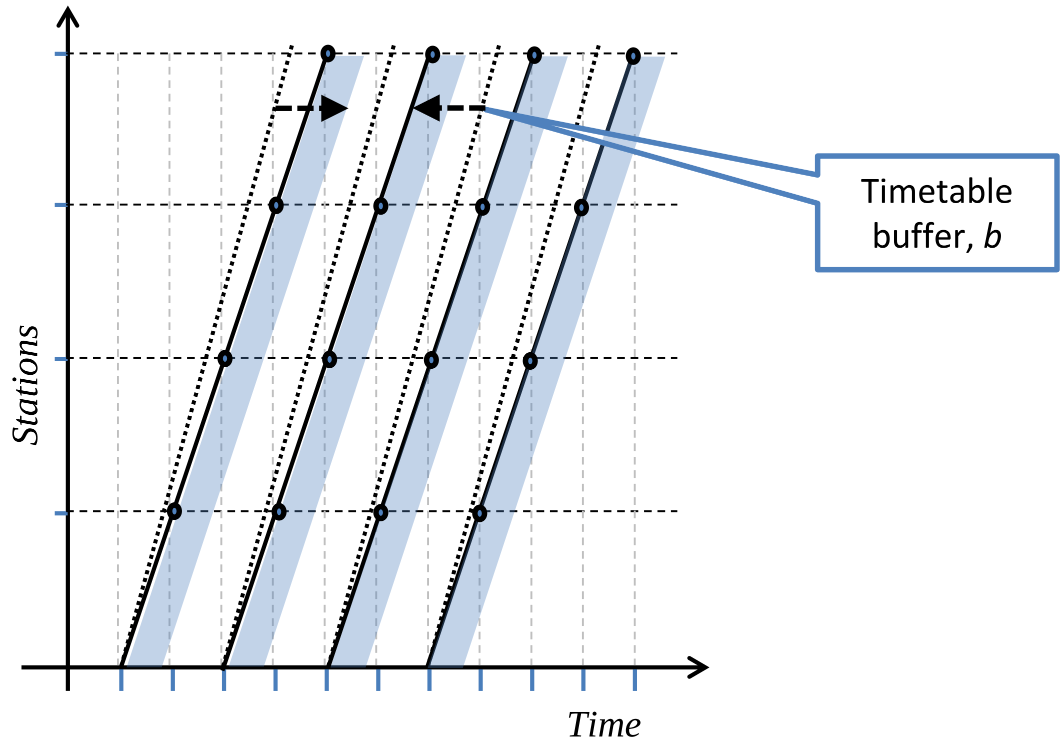
Key Performance Measure: Aggregate Delay

- The timetable is a system
- Punctuality is a systemwide measure
- Measure delayed passenger minutes
- Flawed but convenient equivalent
  - Measure train delays at each station
  - Accuracy depends on homogenous passenger flow at all stations
  - Do not measure train delays at non stopping locations

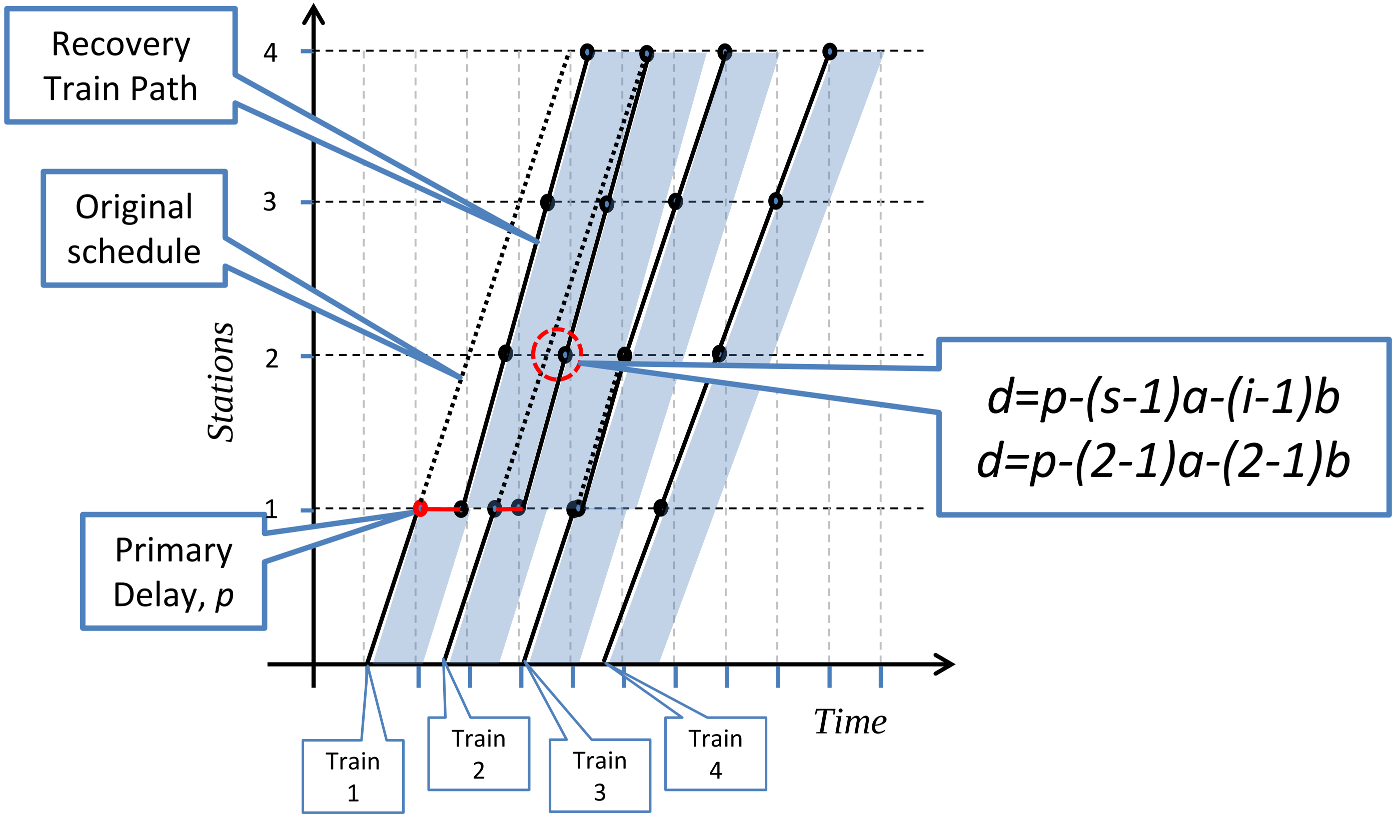
System Definition, Single Train



System Definition, Multiple Trains



Impact of a Primary Delay



Derivation, Bounds of Disruption

$$\Gamma = \sum_{\substack{i \in R \mid d_{i,s} \geq \delta \\ s \in S}} d_{i,s}$$

Solve for: $d_{i,s} \geq \delta$

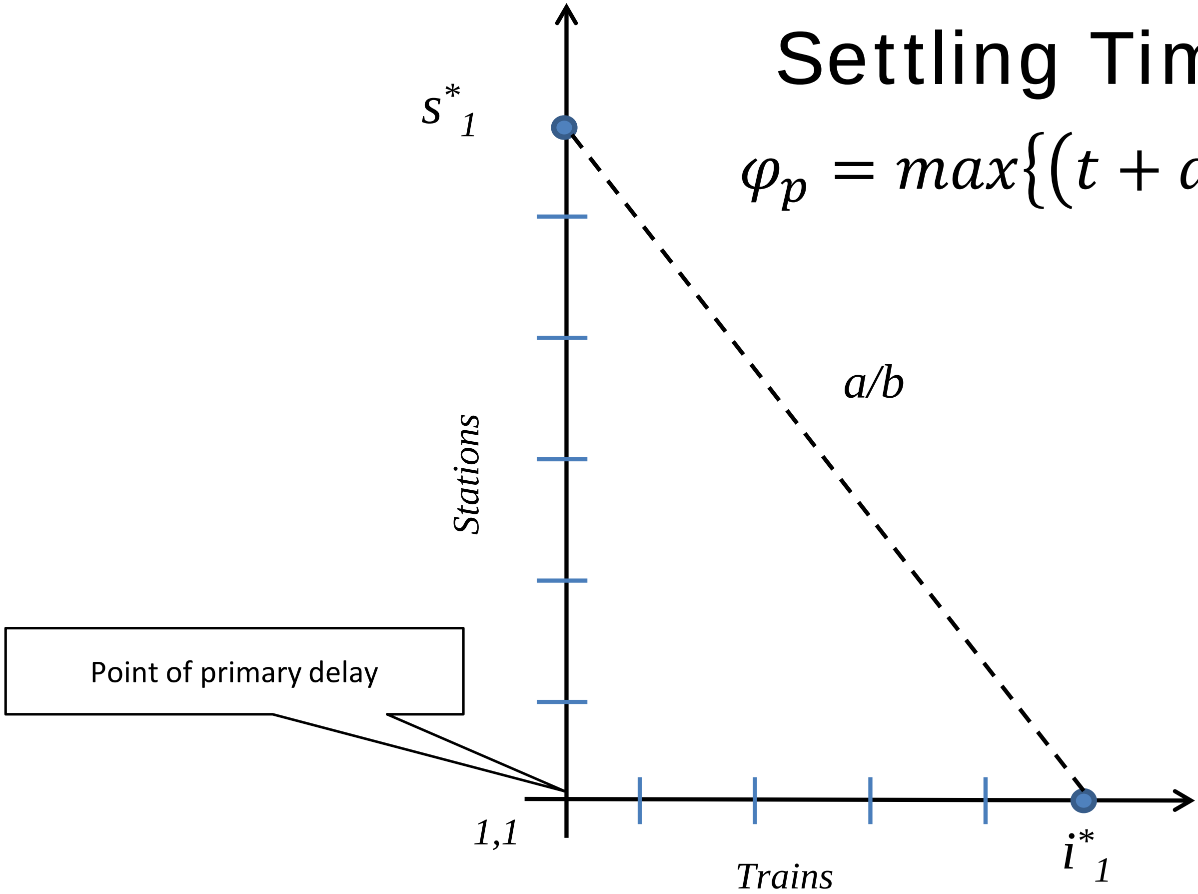
$$s_i^* = \left\lfloor \frac{p + b - \delta}{a} - i\,\frac{b}{a} \right\rfloor + 1 \mid p \geq a + \delta$$

$$i_s^* = \left\lfloor \frac{p + a - \delta}{b} - s\,\frac{a}{b} \right\rfloor + 1 \mid p \geq b + \delta$$

Recovery Region

Settling Time

$$\varphi_p = max\{(t+a)s_1^*, (h+b)i_1^*\}$$



Symmetric System, c=a=b

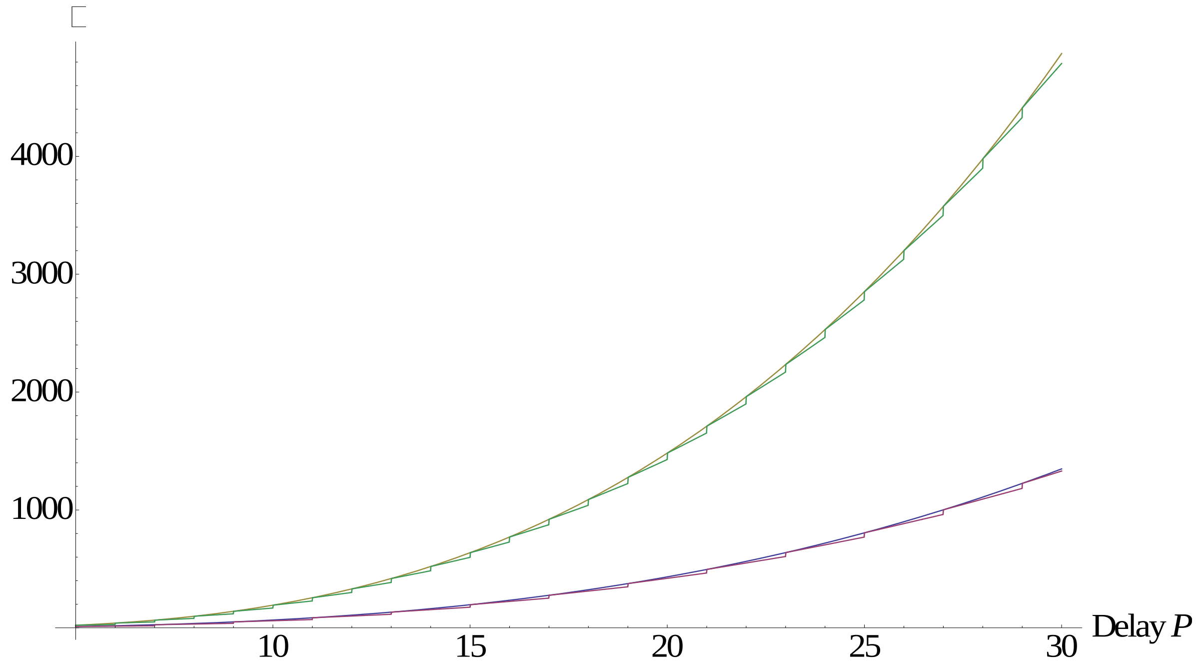
$$s_i^* = \left\lfloor \frac{p-\delta}{c} \right\rfloor - i + 2 \mid p \geq c + \delta$$

$$i_s^* = \left\lfloor \frac{p-\delta}{c} \right\rfloor - s + 2 \mid p \geq c + \delta$$

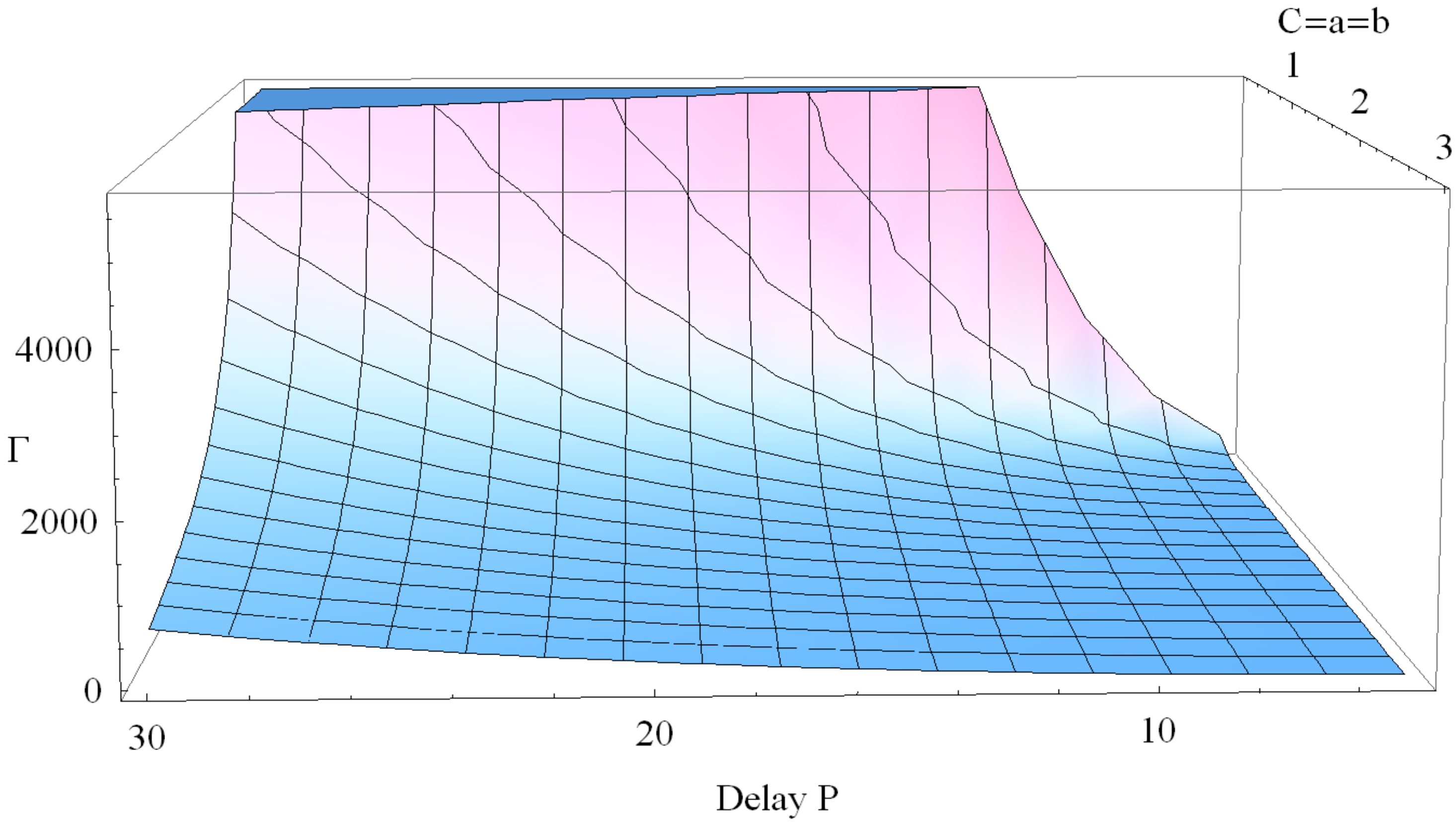
$$\Gamma_s = \sum_{\substack{i \in \left\{1,2,\ldots,\left\lfloor \frac{p-\delta}{c} \right\rfloor - s + 2\right\} \\ s \in \left\{1,2,\ldots \left\lfloor \frac{p-\delta}{c} \right\rfloor + 1\right\}}} p + 2c - c(s+i)$$

Relaxed Floor Generates Polynomial Cumulative Delay

$$\Gamma_s = \frac{p^3}{6c^2} + \frac{p^2}{2c} + \frac{(2c^2 + 3c\delta - 3\delta^2)p}{6c^2} + \frac{4c^2\delta - 6c\delta^2 + 2\delta^3}{6c^2}$$



Visualizing the Polynomial



Settling Time Function φ

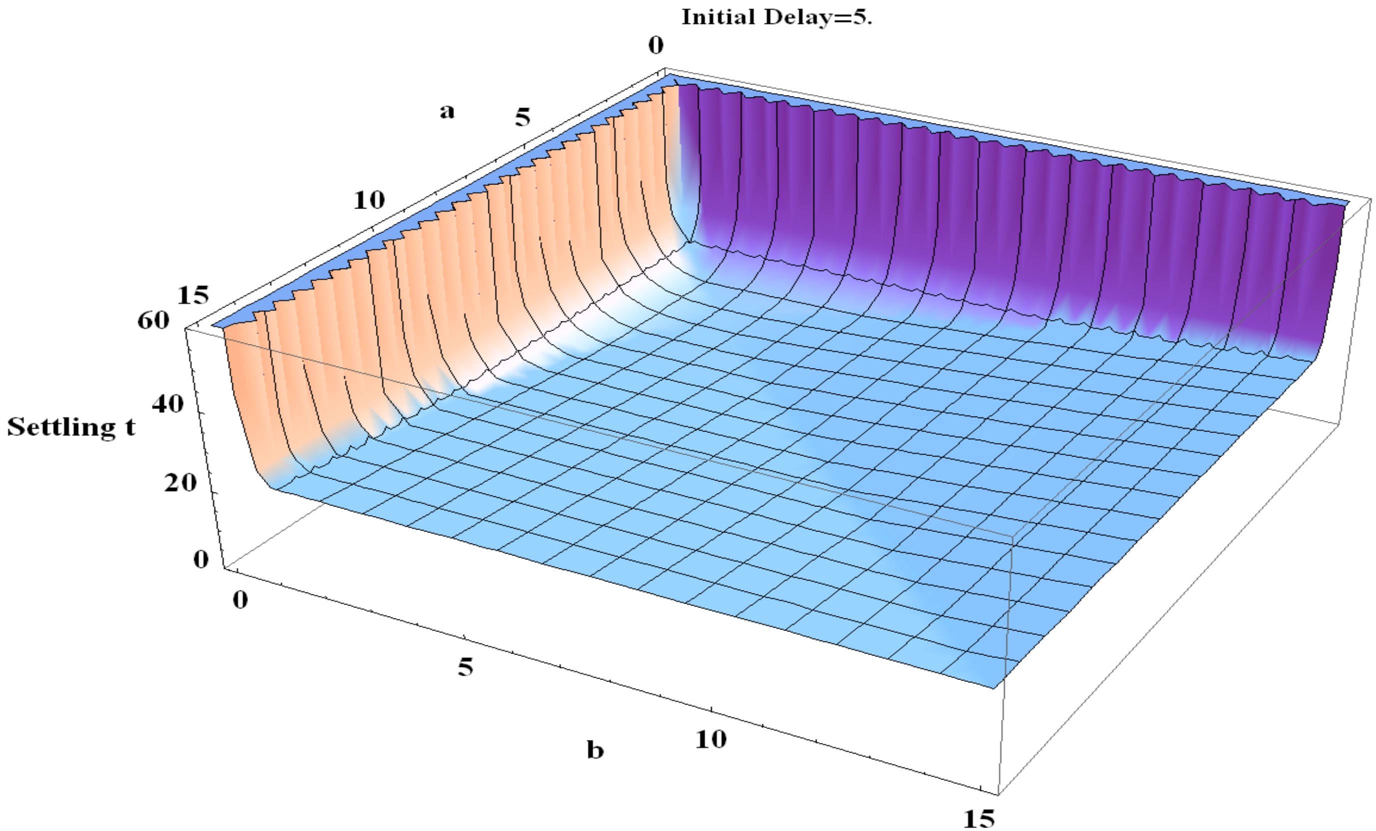
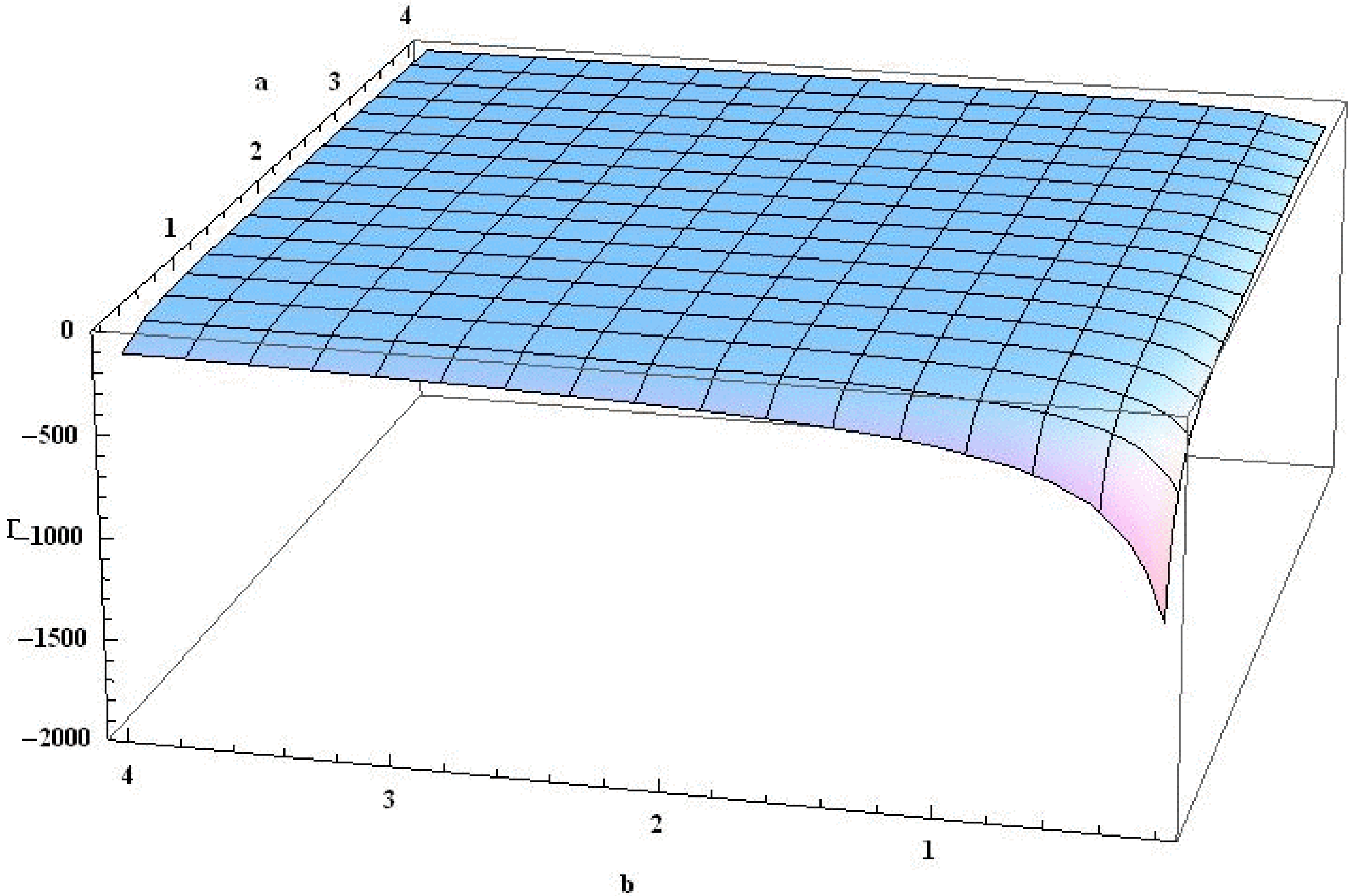


Figure 3-4: Contour of settling time with t=5, h=5, δ=3 and p=5.

Generic Polynomial, p: {5, 20}



Conclusions

- Cumulative train delay at stations is a key performance measure
- Polynomial function is a practical estimate of system delay
- Timetable supplement and timetable buffer should be equal
- Decreasing marginal benefit of increasing supplement/buffer

Tak for i dag!